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DIAGNOSIS OF RHEOLOGICAL PROPERTIES OF VISCOELASTIC–PLASTIC MEDIA DURING THEIR FLOW IN PIPES

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A method is proposed for diagnosing the viscoelastic–plastic properties of a medium.

Methods are available [1-3] for determining the relaxation properties of viscoelastic–plastic media, but the models used in these papers are generally not justified. Because of the complexity of the internal structure of such media, this justification is very difficult. Therefore, methods permitting first a reliable diagnosis of the internal structure of such media and then a determination of their parameters are of important theoretical and practical interest for the analysis and optimization of technological processes related to the flow of viscoelastic–plastic media in pipes and channels.

1. Let us consider pipe flow of a viscoelastic–plastic medium whose rheological equation is described by the following model:

$$\theta \frac{\partial \tau}{\partial t} + \tau - \tau_0 = \mu \left(\frac{\partial V}{\partial r} + \lambda \frac{\partial^2 V}{\partial t \partial r} \right).$$

In writing this relation it is assumed that the velocity gradient and the stress are stabilized along the length.

It should be noted that this model was employed in [4] to describe bituminous mineral conglomerates such as asphalt concretes and their components.

The differential equations of motion for a viscoelastic–plastic medium in an elastic pipe have the form [1]

$$\left.\begin{aligned} \theta \frac{\partial^2 W}{\partial t^2} + (1 + 2a\lambda) \frac{\partial W}{\partial t} + 2aW + \frac{2\tau_0}{R} &= -\frac{1}{\rho} \left(\frac{\partial P}{\partial z} + \theta \frac{\partial^2 P}{\partial t \partial z} \right), \\ \rho c^2 \frac{\partial W}{\partial z} &= -\frac{\partial P}{\partial t}, \end{aligned}\right\} \quad (1)$$

$$2a = 8\mu/\rho R^2.$$

The diagnosis of the viscoelastic–plastic properties of a medium is based on the solution of the differential equations (1) or, under certain assumptions, on the solution of the first of these equations. By using calculated moments the dependence of certain relations of the moments type on parameters characterizing the viscoelastic properties of the medium can be written in analytic form, and a harmonic analysis can be made.

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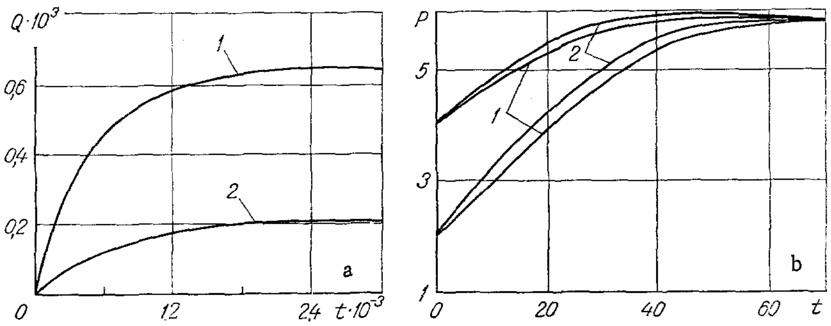

Fig. 1. Dependence of a) flow rate Q (m^3/sec) and b) pressure P (atm) on time t (sec) at (1) 40°C and (2) 60°C.

2. Suppose a viscoelastic—plastic medium flows along the z axis in a pipe. We assume that at $t=0$ the medium is at rest. At a certain instant the pressure drop along the pipe axis is changed abruptly to the constant value $(P_0 - P_1)/L$. We assume that the velocity remains constant along the length of the pipe and varies according to a known law. Under these assumptions the motion of the medium is described by the first of Eqs. (1) whose solution for the initial conditions

$$W(0)=0,\ \frac{dW(0)}{dt}=0$$

has the form

$$W=\frac{P_0-P_1-\dfrac{2\tau_0 L}{R}}{2a\rho L}\left(1+\frac{k_2}{k_1-k_2}e^{k_1 t}-\frac{k_1}{k_1-k_2}e^{k_2 t}\right), \tag{2}$$

$$k_{1,2}=\frac{-(1+2a\lambda)\pm\sqrt{(1+2a\lambda)^2-8a\theta}}{2\theta}.$$

For $t\to\infty$ $W_\infty=\dfrac{P_0-P_1-\dfrac{2\tau_0 L}{R}}{2a\rho L}$.

The zero and first moments are calculated with the formulas

$$w_i=\int_0^\infty [W_\infty - W(t)]\, t^i dt,\quad i=0,1. \tag{3}$$

By substituting (2) into (3) and making some transformations, we obtain

$$w_0=\frac{P_0-P_1-\dfrac{2\tau_0 L}{R}}{2a\rho L}\,\frac{1+2a\lambda}{2a},$$

$$w_1=\frac{P_0-P_1-\dfrac{2\tau_0 L}{R}}{2a\rho L}\left[\left(\frac{1+2a\lambda}{2a}\right)^2-\frac{\theta}{2a}\right].$$

To diagnose the viscoelastic—plastic properties of the medium, we introduce the following relations:

$$\frac{w_1}{w_0}=\frac{1+2a\lambda}{2a}-\frac{\theta}{1+2a\lambda}, \tag{4}$$

$$\frac{w_0}{W_\infty}=\frac{1+2a\lambda}{2a}, \tag{5}$$

$$\frac{W_\infty \rho L}{P_0-P_1-\dfrac{2\tau_0 L}{R}}=\frac{1}{2a}. \tag{6}$$

TABLE 1. Results of First Series of Experiments

τ_{01}, $N\cdot m^{-2}$	$2a$, sec^{-1}	μ, $N\cdot sec\cdot m^{-2}$	w_0, m	w_1, m·sec	$\frac{W_\infty \rho L}{P_0-P_1-\frac{2\tau_0 L}{R}}$, sec	$\frac{w_1}{w_0}$, sec	$\frac{w_0}{W_\infty}$, sec	λ, sec	θ, sec
33	6	0,37	109	37455	0,17	346	346	346	0
33	6	0,37	37	14349	0,17	381	381	381	0

Analysis of (4)-(6) shows that the following conditions can be satisfied:

$$\text{I.}\quad \frac{w_1}{w_0}=\frac{w_0}{W_\infty}=\frac{W_\infty \rho L}{P_0-P_1-\dfrac{2\tau_0 L}{R}}=\frac{1}{2a}\quad (\theta=0,\ \ \lambda=0),$$

$$\text{II.}\quad \frac{w_1}{w_0}\neq\frac{w_0}{W_\infty}=\frac{W_\infty \rho L}{P_0-P_1-\dfrac{2\tau_0 L}{R}}=\frac{1}{2a}\quad (\theta\neq 0,\ \ \lambda=0),$$

$$\text{III.}\quad \frac{w_1}{w_0}=\frac{w_0}{W_\infty}\neq\frac{W_\infty \rho L}{P_0-P_1-\dfrac{2\tau_0 L}{R}}=\frac{1}{2a}\quad (\theta=0,\ \ \lambda\neq 0),$$

$$\text{IV.}\quad \frac{w_1}{w_0}\neq\frac{w_0}{W_\infty}\neq\frac{W_\infty \rho L}{P_0-P_1-\dfrac{2\tau_0 L}{R}}=\frac{1}{2a}\quad (\theta\neq 0,\ \ \lambda\neq 0).$$

Using the law of variation of the velocity for a constant pressure drop, the moments are calculated with Eq. (3), and then the ratios w_1/w_0, w_0/W_∞, $W_\infty \rho L/(P_0 - P_1 - 2\tau_0 L/R)$ (τ_0 and $2a$ are determined from Eq. (6) for two steady flow conditions), which must satisfy one of the conditions I-IV. Knowing the values of these ratios, we determine the viscoelastic properties of the medium from Eqs. (4) and (5).

To test the proposed method of diagnosis, we consider the flow of a high-viscosity oil with a density $\rho = 0.8$ g/cm^3 in a pipeline 630 m long and 2 in in diameter (experimental data taken from [1]). Figure 1a shows curves for the flow rate at constant pressure $P_1 = 0$ at the outlet and $P_0 = 26.1$ atm (curve 1) and $P_0 = 19.6$ atm (curve 2) at the inlet. Table 1 shows the calculated results. It is clear from the table that this oil satisfies condition III, i.e., it is a viscoelastic—plastic medium described by the model

$$\tau-\tau_0=\mu\left(\frac{\partial V}{\partial r}+\lambda\frac{\partial^2 V}{\partial t\partial r}\right).$$

3. It is assumed that at $t = 0$ the medium is moving with the average velocity W_0. From $t = 0$ the pressure drop is varied according to a definite law $\partial P/\rho\partial z = f(t)$, with the time dependence of the velocity of the medium assumed known. The problem reduces to that of solving the first of Eqs. (1) with the initial conditions

$$W(0)=W_0,\quad \frac{dW(0)}{dt}=0. \tag{7}$$

The Laplace transform of the first of Eqs. (1) under conditions (7) is

$$[2a+(1+2a\lambda)s+\theta s^2]W^*-[(1+2a\lambda)+\theta s]W_0+\frac{2\tau_0}{\rho R s}=(1+\theta s)f^*-\theta f_0, \tag{8}$$

where $W^*(s)=\int\limits_0^\infty W(t)e^{-st}dt;\ \ f^*(s)=\int\limits_0^\infty f(t)e^{-st}dt; f_0=f(0)$.

Using the series expansion of $\exp(-st)$, it is easy to show that

$$W^*-\frac{W_\infty}{s}=\int\limits_0^\infty [W(t)-W_\infty]e^{-st}dt=w_0-sw_1+\frac{s^2}{2!}w_2-\ldots, \tag{9}$$

TABLE 2. Results of Second Series of Experiments

T, °C	τ_0, N·m^{-2}	$2a$, sec^{-1}	P_1^0, atm·sec	P_2^0, atm·sec	P_1^1, atm·sec^2	P_2^1, atm·sec^2	L.H.S. (5·10), sec^{-1}	L.H.S. (5·11), sec^{-1}	θ, sec	λ, sec
40	0	11000	—33,3	—64,0	—451,4	—897,7	$9{,}723\cdot10^6$	$6{,}425\cdot10^6$	15	409
60	0	8900	—27,6	—55,6	—306	—692	$8{,}868\cdot10^6$	$6{,}378\cdot10^6$	14	213

$$f^* - \frac{f_\infty}{s} = \int_0^\infty [f(t) - f_\infty]\, e^{-st}dt = F_0 - sF_1 + \frac{s^2}{2!}F_2 - \ldots, \tag{10}$$

where $w_i = \int_0^\infty [W(t) - W_\infty]\, t^i dt;\ \ F_i = \int_0^\infty [f(t) - f_\infty]\, t^i dt; f_\infty = f(\infty); i = 0, 1, 2, \ldots$

By substituting (9) and (10) into (8), making some transformations, and equating coefficients of identical powers of s, we obtain

$$2aW_\infty = f_\infty - \frac{2\tau_0}{\rho R}, \tag{11}$$

$$2aw_0 + (1 + 2a\lambda)(W_\infty - W_0) = \theta(f_\infty - f_0) + F_0, \tag{12}$$

$$\theta(W_\infty - W_0) + (1 + 2a\lambda)\, w_0 - 2aw_1 = \theta F_0 - F_1. \tag{13}$$

We transform (11)-(13) by introducing the following relations:

$$\frac{f_\infty - \dfrac{2\tau_0}{\rho R}}{W_\infty} = 2a, \tag{14}$$

$$\frac{F_1 W_\infty (W_\infty - W_0) + F_0 w_0 W_\infty - \left(f_\infty - \dfrac{2\tau_0}{\rho R}\right) w_0^2}{W_\infty w_1 (W_\infty - W_0)} = 2a - \theta\, \frac{(f_\infty - f_0)\, w_0 + (W_\infty - W_0 - F_0)(W_\infty - W_0)}{w_1 (W_\infty - W_0)}, \tag{15}$$

$$\frac{F_1 + w_0}{w_1} = 2a - 2a\lambda\frac{w_0}{w_1} - \theta\, \frac{W_\infty - W_0 - F_0}{w_1}. \tag{16}$$

From the relations obtained, the following conditions can be satisfied:

$$\text{I.}\ \ \frac{F_1 + w_0}{w_1} = \frac{F_1 W_\infty (W_\infty - W_0) + F_0 W_\infty w_0 - \left(f_\infty - \dfrac{2\tau_0}{\rho R}\right) w_0^2}{W_\infty w_1 (W_\infty - W_0)} = \frac{f_\infty - \dfrac{2\tau_0}{\rho R}}{W_\infty} = 2a \quad (\theta = 0,\ \lambda = 0),$$

$$\text{II.}\ \ \frac{F_1 + w_0}{w_1} \neq \frac{F_1 W_\infty (W_\infty - W_0) + F_0 W_\infty w_0 - \left(f_\infty - \dfrac{2\tau_0}{\rho R}\right) w_0^2}{W_\infty w_1 (W_\infty - W_0)} = \frac{f_\infty - \dfrac{2\tau_0}{\rho R}}{W_\infty} = 2a \quad (\theta = 0,\ \lambda \neq 0),$$

$$\text{III.}\ \ \frac{F_1 + w_0}{w_1} \neq \frac{F_1 W_\infty (W_\infty - W_0) + F_0 W_\infty w_0 - \left(f_\infty - \dfrac{2\tau_0}{\rho R}\right) w_0^2}{W_\infty w_1 (W_\infty - W_0)} \neq \frac{f_\infty - \dfrac{2\tau_0}{\rho R}}{W_\infty} = 2a \quad \begin{pmatrix} \theta \neq 0,\ \lambda = 0 \\ \text{or} \\ \theta \neq 0,\ \lambda \neq 0. \end{pmatrix}.$$

Processing the curves shown in Fig. 1a by formulas (14)-(16) gave the same results as in the preceding paragraph. Thus, we can assume that the methods proposed are satisfactory for the diagnosis of the relaxation properties of viscoelastic—plastic media.

4. Suppose a pipeline has been operating for a long time in a periodic regime as a result of a harmonic variation of the pressure drop. Then the pressure drop can be written in the form

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} = \frac{P_0 - P_1}{\rho L} = A_0 + A_1 \cos\omega t. \tag{17}$$

The solution of the first of Eqs. (1) under condition (17) has the form

$$W = \frac{A_0}{2a} - \frac{2\tau_0}{2a\rho L} + \frac{A_1}{\omega|Z_m|}\sin(\omega t - \varphi + \psi), \tag{18}$$

$$|Z_m| = \left[(1 + 2a\lambda)^2 + \left(\theta\omega - \frac{2a}{\omega}\right)^2 \right]^{1/2},$$

$$\sin\varphi = \frac{\theta\omega - \frac{2a}{\omega}}{|Z_m|^{1/2}}, \quad \cos\varphi = \frac{1 + 2a\lambda}{|Z_m|^{1/2}}, \quad \sin\psi = \frac{\theta\omega}{|Z_m|^{1/2}}, \quad \cos\psi = \frac{1}{|Z_m|^{1/2}}.$$

The parameters $2a$ and τ_0 are determined under two flow conditions corresponding to a stationary velocity distribution.

The relaxation parameters θ and λ are diagnosed by using the following relations obtained from the harmonic component of (18) under the assumption that $\theta\omega \ll 2a/\omega$:

$$|Z_m| \sin(\varphi - \psi) = -\left(\frac{2a}{\omega} + 2a\lambda\theta\omega\right), \tag{19}$$

$$|Z_m| \cos(\varphi - \psi) = 1 + 2a(\lambda - \theta). \tag{20}$$

The calculations of the left-hand sides of Eqs. (19) and (20) are based on the processing of pressure and velocity data. An analysis of Eqs. (19) and (20) shows that

$$\text{I. } |Z_m| \sin(\varphi - \psi) = -\frac{2a}{\omega}, \ |Z_m| \cos(\varphi - \psi) = 1 \quad (\theta = 0, \ \lambda = 0),$$

$$\text{II. } |Z_m| \sin(\varphi - \psi) = -\frac{2a}{\omega}, \ |Z_m| \cos(\varphi - \psi) > 1 \quad (\theta = 0, \ \lambda \neq 0),$$

$$\text{III. } |Z_m| \sin(\varphi - \psi) = -\frac{2a}{\omega}, \ |Z_m| \cos(\varphi - \psi) < 1 \quad (\theta \neq 0, \ \lambda = 0),$$

$$\text{IV. } |Z_m| \sin(\varphi - \psi) \neq -\frac{2a}{\omega}, \ |Z_m| \cos(\varphi - \psi) \gtrless 1 \quad (\theta \neq 0, \ \lambda \neq 0).$$

By solving Eqs. (19) and (20) for λ and θ, we obtain

$$\theta = \left\{ -[|Z_m| \cos(\varphi - \psi) - 1]\,\omega + \sqrt{[|Z_m| \cos(\varphi - \psi) - 1]^2 \omega^2 - 4 \cdot 2a\omega \left[|Z_m| \sin(\varphi - \psi) + \frac{2a}{\omega} \right]} \right\} \{2 \cdot 2a\omega\}^{-1}, \tag{21}$$

$$\lambda = \frac{|Z_m| \cos(\varphi - \psi) - 1 + 2a\theta}{2a}. \tag{22}$$

It is clear from (21) and (22) that the accuracy of the relaxation parameters being diagnosed and determined depends on the amplitude and phase shift of the signal. Therefore, if there is considerable noise in the measured data, it is necessary to employ statistical methods of frequency characteristics [5]. We propose a method for computing the amplitude and phase shift of the signal when noise is present in the measured data. It is assumed that the periodic component of the pressure drop and the velocity can be represented as the sum of a harmonic oscillation and noise:

$$-\frac{1}{\rho}\left(\frac{\partial P}{\partial z}\right)_N = \Delta P = A_1 \cos\omega t + \varepsilon_1(t), \quad W_N = \frac{A_1}{\omega |Z_m|} \sin(\omega t - \varphi + \psi) + \varepsilon_2(t),$$

where $\varepsilon_1(t)$ and $\varepsilon_2(t)$ represent stationary random noise with $M[\varepsilon_1(t)] = M[\varepsilon_2(t)] = 0$. Then by following [6], the following relations are calculated:

$$R_{12} = \lim_{T \to \infty} \frac{1}{T} \int_0^T W_N(t)\, \Delta P(t)\, dt = -\frac{A_1^2}{2\omega |Z_m|} \sin(\varphi - \psi), \tag{23}$$

$$R_{13} = \lim_{T \to \infty} \frac{1}{T} \int_0^T W_N(t)\, \Delta P_0(t)\, dt = \frac{A_1^2}{2\omega |Z_m|} \cos(\varphi - \psi), \tag{24}$$

$$R_{11} = \lim_{T \to \infty} \frac{1}{T} \int_0^T W_N(t)\, W_N(t)\, dt = \frac{A_1^2}{2\omega^2 |Z_m|^2}, \tag{25}$$

where $\Delta P_0(t) = A_1 \sin\omega t$, R_{12} and R_{13} are cross-correlation functions, and R_{11} is an autocorrelation function.

The right-hand sides of the last equations are functions of the parameters of the problem, and the left-hand sides are determined from experimental data. It follows from (23), (24), and (25) that

$$A_1 = \sqrt{\frac{2(R_{12}^2 + R_{13}^2)}{R_{11}}}, \quad \mathrm{tg}(\varphi - \psi) = -\frac{R_{12}}{R_{13}}, \ |Z_m| = \frac{A_1}{2\omega\sqrt{R_{12}^2 + R_{13}^2}}.$$

By using Eqs. (19) and (20) and the formulas derived, the viscoelastic properties of the medium can be diagnosed.

Since there are no data, a mathematical experiment was performed to diagnose viscoelastic properties during the periodic motion of media in pipes. The input data were taken as follows: $\tau_0 = 500$ mg/cm^2, $2a = 1$ sec^{-1}, $\lambda = 500$ sec, $\theta = 100$ sec, $\omega = 10^{-3}$ sec^{-1}, $\rho = 1$ g/cm^3, $L = 1000$ m, $A_0 = 1$ m/sec, $A_1 = 0.5$ m/sec.

After the values of the velocity were calculated with (18), the values of the pressures and velocities were obtained with an error of no more than 10%. By processing the distorted pressure and velocity curves the values $\theta = 124$ sec and $\lambda = 604$ sec for the relaxation parameters were found from Eqs. (21) and (22). The values of these quantities found by a statistical calculation were $\theta = 105$ sec, $\lambda = 515$ sec.

Thus, the calculations performed show that the proposed method of diagnosing and determining the relaxation parameters of a medium is in good agreement with the input data.

5. After a constant flow rate of the viscoelastic—plastic medium was established, the pipe outlet was closed, and pressure curves were measured at both inlet and outlet. In this case the process is described by Eqs. (1) with the following initial and boundary conditions:

$$W(0, z) = W_0, \ P(0, z) = P_{10} - \frac{P_{10} - P_{20}}{L} z, \ \left.\frac{\partial^2 P}{\partial t^2}\right|_{t=0} = 0,$$
$$P(t, 0) = P_1(t), \ P(t, L) = P_2(t).$$

The supplementary condition $W(t, L) = 0$ is specified for the diagnosis and determination of the relaxation properties of the medium. By solving (1) for $P(t, z)$, we obtain

$$\theta \frac{\partial^3 P}{\partial t^3} + (1 + 2a\lambda)\frac{\partial^2 P}{\partial t^2} + 2a\frac{\partial P}{\partial t} = c^2\left(\theta \frac{\partial^3 P}{\partial t \partial z^2} + \frac{\partial^2 P}{\partial z^2}\right). \tag{26}$$

The initial and boundary conditions for $P(t, z)$ are written in the form

$$P(0, z) = P_{10} - \frac{P_{10} - P_{20}}{L} z, \ \left.\frac{\partial P}{\partial t}\right|_{t=0} = 0, \ \left.\frac{\partial^2 P}{\partial t^2}\right|_{t=0} = 0, \tag{27}$$

$$P(t, 0) = P_1(t), \ P(t, L) = P_2(t). \tag{28}$$

The supplementary boundary condition takes the form

$$\left.\left(\theta \frac{\partial^2 P}{\partial t \partial z} + \frac{\partial P}{\partial z}\right)\right|_{z=L} = -\frac{2\tau_0}{R}. \tag{29}$$

Taking account of (27) and (28), the Laplace transform of (26) is

$$P^*(s, z) = C_1 e^{\alpha z} + C_2 e^{-\alpha z} + \frac{1}{s}\left(P_{10} - \frac{P_{10} - P_{20}}{L} z\right), \tag{30}$$

$$C_1 = \frac{P_2^*(s) - P_1^*(s)e^{-\alpha L} - \frac{P_{10}}{s}(1 - e^{-\alpha L}) + \frac{P_{10} - P_{20}}{s}}{e^{\alpha L} - e^{-\alpha L}},$$

$$C_2 = \frac{-P_2^*(s) + P_1^*(s)e^{\alpha L} + \frac{P_{10}}{s}(1 - e^{\alpha L}) - \frac{P_{10} - P_{20}}{s}}{e^{\alpha L} - e^{-\alpha L}},$$

$$\alpha^2 = \frac{s[2a + s(1 + 2a\lambda) + s^2\theta]}{c^2(1 + s\theta)},$$

$$P^*(s, z) = \int_0^\infty P(t, z) e^{-st} dt; \; P^*_{1,2}(s) = \int_0^\infty P_{1,2}(t) e^{-st} dt.$$

Using (30), the transform of the supplementary boundary condition (29) can be written as

$$\left.\frac{\partial P^*}{\partial z}\right|_{z=L} = \frac{P_2^*(e^{\alpha L} + e^{-\alpha L}) - 2P_1^* - \frac{P_{20}}{s}(e^{\alpha L} + e^{-\alpha L}) + 2\frac{P_{10}}{s}}{e^{\alpha L} - e^{-\alpha L}} = \frac{\frac{P_{10} - P_{20}}{L} - \frac{2\tau_0}{R}}{\alpha s(1 + s\theta)}. \tag{31}$$

We introduce the k-th order moments of the functions $P_i(t) - P_{i\infty}$ in the form [7]

$$P_i^k = \int_0^\infty [P_i(t) - P_{i\infty}] \frac{t^k}{k!} dt, \; P_{i\infty} = \lim_{t \to \infty} P_i(t), \; i = 1,2, \; k = 0, 1, 2, \ldots$$

The following relations hold:

$$P_i^* = \frac{P_{i\infty}}{s} + P_i^0 - sP_i^1 + s^2 P_i^2 - \ldots \tag{32}$$

Using the series expansion of $\exp(\alpha L)$ and limiting ourselves to six or seven terms (since αL is small), making some transformations, using (32), and equating coefficients of identical powers of s, we obtain the following relations:

$$P_{1\infty} - P_{2\infty} = \frac{2\tau_0 L}{R}, \tag{33}$$

$$\frac{P_{10} - P_{20} - P_{1\infty} + P_{2\infty}}{\rho L W_0} = 2a, \tag{34}$$

$$6\frac{P_1^0 - P_2^0}{2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10}} \frac{c^2}{L^2} = 2a + 6\theta \frac{P_{2\infty} - P_{20} - P_{1\infty} + P_{10}}{2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10}} \frac{c^2}{L^2}, \tag{35}$$

$$\begin{aligned} 20\frac{c^2}{L^2} & \frac{P_1^1 - P_2^1 + \frac{L^2}{6c^2}(2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10})}{\frac{L^2}{3c^2}(P_{2\infty} - P_{20})\frac{P_{2\infty} - P_{1\infty} - P_{20} + P_{10}}{\rho L W_0} - P_1^0 - 9P_2^0} = 2a \\ & + 20\theta\frac{c^2}{L^2} \frac{P_1^0 - P_2^0 - \frac{7}{60}\frac{2aL^2}{c^2}(P_{2\infty} - P_{20} - P_{1\infty} + P_{10})}{\frac{L^2}{3c^2}(P_{2\infty} - P_{20})\frac{P_{2\infty} - P_{1\infty} - P_{20} + P_{10}}{\rho L W_0} - P_1^0 - 9P_2^0} \\ & - \frac{10}{3} 2a\lambda \frac{2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10}}{\frac{L^2}{3c^2}(P_{2\infty} - P_{20})\frac{P_{2\infty} - P_{1\infty} - P_{20} + P_{10}}{\rho L W_0} - P_1^0 - 9P_2^0}. \end{aligned} \tag{36}$$

Equation (34) was derived on the assumption that the velocity of the medium for steady motion corresponds to the relation

$$W_0 = \frac{P_{10} - P_{20}}{2a\rho L} - \frac{2\tau_0}{2a\rho R}.$$

The limiting shear stress of the liquid τ_0 can be determined from (33). Analysis of (34), (35), and (36) shows that if the left-hand sides of these equations are equal to one another, $\theta = 0$ and $\lambda = 0$. If this is not the case, $\theta \neq 0$ and $\lambda \neq 0$, or $\theta \neq 0$ and $\lambda = 0$. When the left-hand side of (35) is equal to the left-hand side of (34), but not equal to the left-hand side of (36), $\theta = 0$ and $\lambda \neq 0$. The coefficient of friction $2a$ is found from (34). The relaxation times are calculated with formulas obtained from (35) and (36):

$$\theta = \frac{P_1^0 - P_2^0 - \frac{2aL^2}{6c^2}(2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10})}{P_{2\infty} - P_{20} - P_{1\infty} + P_{10}},$$

$$\lambda = \frac{\frac{2aL^2}{20c^2}\left[\frac{L^2}{3c^2}(P_{2\infty} - P_{20})\frac{P_{2\infty} - P_{20} - P_{1\infty} + P_{10}}{\rho L W_0} - P_1^0 - 9P_2^0\right] +}{\frac{2aL^2}{6c^2}(2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10})} \rightarrow$$

$$\rightarrow \frac{+\theta\left|P_1^0 - P_2^0 - \frac{7}{60}\frac{2aL^2}{c^2}(P_{2\infty} - P_{20} - P_{1\infty} + P_{10}) - \right.}{} \rightarrow$$

$$\rightarrow \frac{-P_1^1 + P_2^1 - \frac{L^2}{6c^2}(2P_{2\infty} - 2P_{20} + P_{1\infty} - P_{10})}{}.$$

The pressure recovery curves [1] on a capillary of length L = 160 cm, radius R = 0.2 cm, with high-viscosity oil shown in Fig. 1b were processed by the proposed method of diagnosis.

Table 2 lists the results of the calculations. It is clear from the table that this oil is a viscoelastic medium described by the model

$$\theta \frac{\partial \tau}{\partial t} + \tau = \mu\left(\frac{\partial V}{\partial r} + \lambda \frac{\partial^2 V}{\partial t \partial r}\right).$$

In conclusion, it should be noted that the relaxation properties of viscoelastic—plastic media can also be diagnosed and estimated on full-scale installations such as pipelines and wells.

NOTATION

τ, shear stress; τ_0, limiting shear stress; $\partial V/\partial r$, velocity gradient; θ, λ, relaxation time; t, time; W, velocity averaged over cross section of pipe; P, pressure; R, pipe radius; c, wave velocity in pipe; A_0, mean pressure drop; A_1, amplitude of oscillation; $\varepsilon_1(t)$, $\varepsilon_2(t)$, stationary random noise; w_0, w_1, F_0, F_1, P_i^k, determined moments of prescribed functions.

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